TRINITY COLLEGE

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

See next page

Question 8 **(8 marks)**

A random sample of adults who were not working and not seeking work were recently asked for the main reason that they were not looking for work. The responses, categorised by the sex of the adult and their main reason, are summarised in the table below.

Reason	Male	Female
Education	143	116
Family considerations	33	117
Other	84	37

(a) How many adults gave a response? (1 mark)

Solution

$143 + 33 + 84 + 116 + 117 + 37 = 530$ adults.

Specific behaviours

✓ correct total

(b) What percentage of the females gave education as their main reason? (2 marks)

Solution

$116 + 117 + 37 = 270$

$116 \div 270 = 43\%$

Specific behaviours

✓ indicates use of correct figures
✓ any percentage that rounds to 43%

(c) Construct a table showing column percentages for the above data, rounding entries to the nearest whole number. (3 marks)

Solution

M_{TOT}: $530 - 270 = 260$, $143 \div 260 = 55\%$, etc

Reason	Male (%)	Female (%)
Education	55	43
Family considerations	13	43
Other	32	14

Specific behaviours

✓ neat table with row and column headings
✓ both columns add to 100
✓ both columns correct

(d) Discuss whether the data from the survey suggests the presence of an association between the variables sex and reason. (2 marks)

Solution

The data does suggest an association exists between the variables as the pairs of percentages in each row (i.e., for each reason) are quite different for males and females.

Specific behaviours

✓ states association exists
✓ explains using difference in percentages across reason categories

See next page

Question 9 **(7 marks)**

(a) The monthly units of electricity u consumed by each apartment in a building was strongly associated with the average monthly maximum temperature, T °C. The least-squares line for the variables was $\hat{u} = 211.5 - 2.8T$.

(i) Predict the units of electricity consumed by an apartment in a month when the average monthly maximum temperature was 13°C. (1 mark)

Solution
$\hat{u} = 211.5 - 2.8(13) = 175.1$ units.
Specific behaviours
✓ correct prediction

(ii) In a month when the average monthly maximum temperature was 24°C, an apartment consumed 146.6 units of electricity. Calculate the residual for this data point. (2 marks)

Solution
$\hat{u} = 211.5 - 2.8(24) = 144.3$ Residual: $u - \hat{u} = 146.6 - 144.3 = 2.3$ units.
Specific behaviours
✓ indicates correct $\hat{u}$ ✓ correct residual

(b) In a government study, the correlation coefficient for the association between age and superannuation balance for employed adults was found to be 0.728. What percentage of the variation in superannuation balance for employed adults is unexplained by their variation in age? (2 marks)

Solution
$r^2 = 0.728^2 = 0.53$ Since 53% of the variation is explained, then 47% is unexplained.
Specific behaviours
✓ calculates coefficient of determination ✓ correct percentage

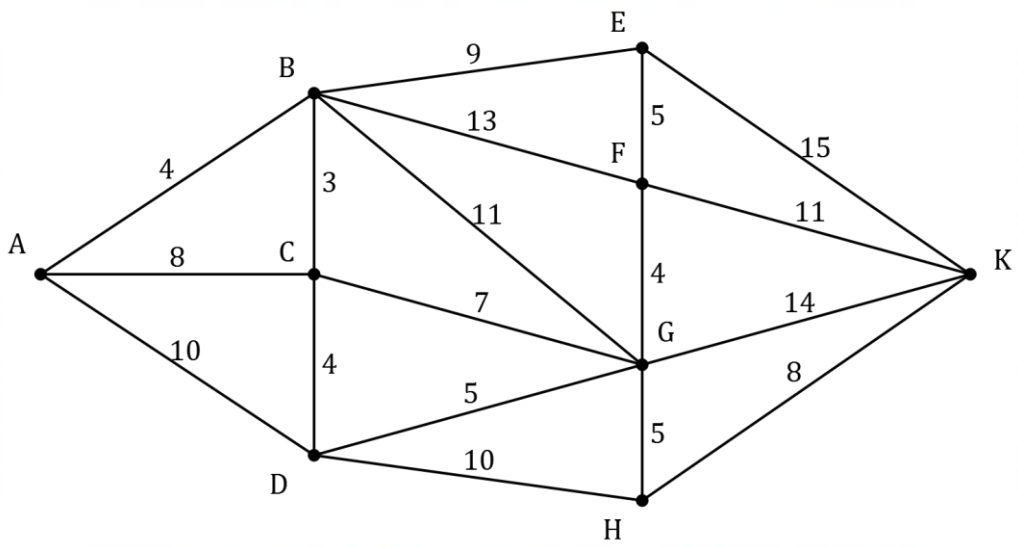
(c) After measuring the age and hearing acuity of a group of pensioners, a researcher observed a negative linear association between the variables and found that 75% of the variation in hearing acuity can be explained by the variation in age. Determine the correlation coefficient for the association. (2 marks)

Solution
$r^2 = 0.75 \rightarrow r = \pm\sqrt{0.75} = \pm 0.866$ Since association is negative, then $r = -0.866$.
Specific behaviours
✓ indicates square root of coefficient of determination ✓ correct correlation coefficient

See next page

Question 10 **(7 marks)**

The graph below represents a network of distribution centres. Each edge weight is the cost in dollars to transport a parcel between adjacent centres (the vertices).



(a) Determine the minimum cost to transport a parcel from A to K and state the path that should be used to achieve this minimum. (3 marks)

Solution
Dijkstra's algorithm (not specified in course - vertex, min. distance from A pairs): $A0, B4, C7, D10, E13, F17, G14, H19, K27$ Hence minimum cost is \$27 using path $A-B-C-G-H-K$.
Specific behaviours
✓ evidence of method (algorithm, listing of trials, etc.) ✓ correct minimum cost ✓ correct path

(b) A new route is proposed between centres C and F which will reduce the minimum cost to transport a parcel from A to K by \$4. Determine the cost to transport a parcel between centres C and F. (2 marks)

Solution
$AC = 7, FK = 11 \rightarrow 7 + CF + 11 = 27 - 4$ $CF = 23 - 18 = 5$ Hence cost is \$5.
Specific behaviours
✓ indicates appropriate method ✓ correct cost

(c) A parcel is transported along a route that is a cycle of 3 edges in the graph. Determine the maximum possible transport cost and describe the corresponding cycle. (2 marks)

Solution
Cycle for maximum cost is $EFKE$ and cost is \$31. (*Also* $EKFE, FEKF$, ... *etc*)
Specific behaviours
✓ correctly describes a cycle ✓ correct cost

See next page

Question 11 **(8 marks)**

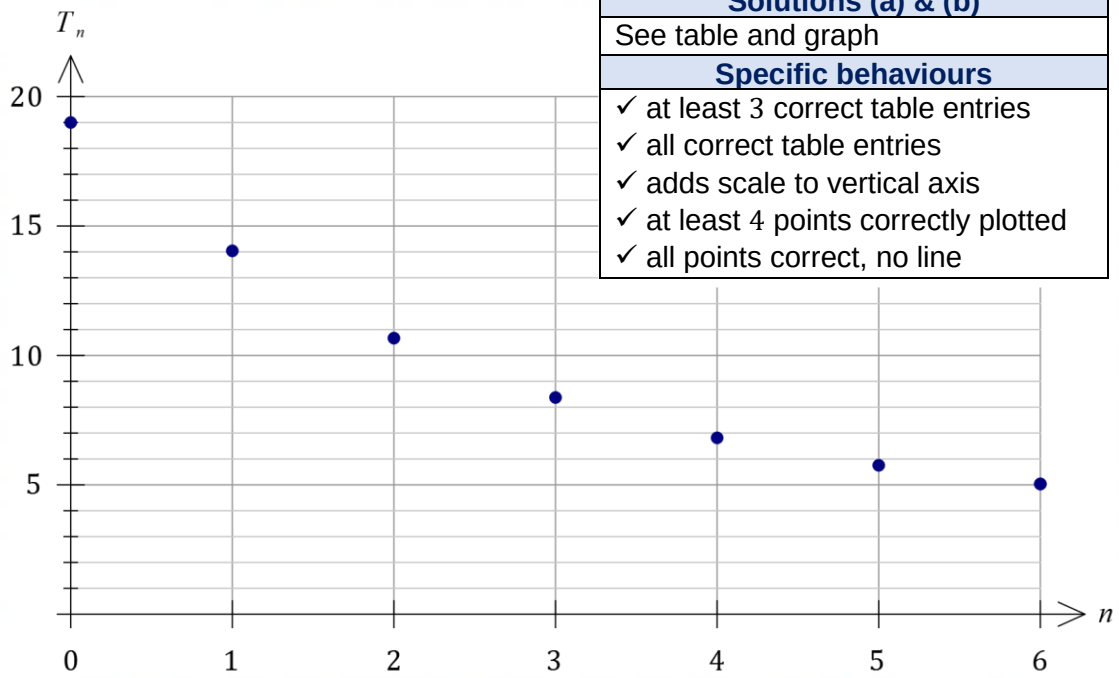
The cooling system for a mobile cool room has just been turned on. The temperature T_n °C inside the cool room, n hours later, is modelled by the linear recurrence relation

$$T_n = 0.68T_{n-1} + 1.12, \qquad T_0 = 19.$$

(a) Complete the table of temperatures below. (2 marks)

n	0	1	2	3	4	5	6
T_n (°C)	19.0	14.0	**10.7**	**8.4**	**6.8**	**5.8**	**5.0**

(b) Add a scale to the vertical axis below and then plot the temperature inside the cool room every hour. (3 marks)



Solutions (a) & (b)
See table and graph
Specific behaviours
✓ at least 3 correct table entries
✓ all correct table entries
✓ adds scale to vertical axis
✓ at least 4 points correctly plotted
✓ all points correct, no line

(c) After how many hours does the model predict that the temperature inside the cool room will first reach within 0.2° of its steady state? Justify your answer. (3 marks)

Solution
Using sequence, steady state temperature is 3.5° C. Hence temperature must fall to 3.7° C or below. From sequence, $T_{11} = 3.72$ and $T_{12} = 3.65$ and so cool room will first reach required temperature after 12 hours.
Specific behaviours
✓ indicates steady state temperature
✓ states correct number of hours
✓ justifies time using terms of sequence

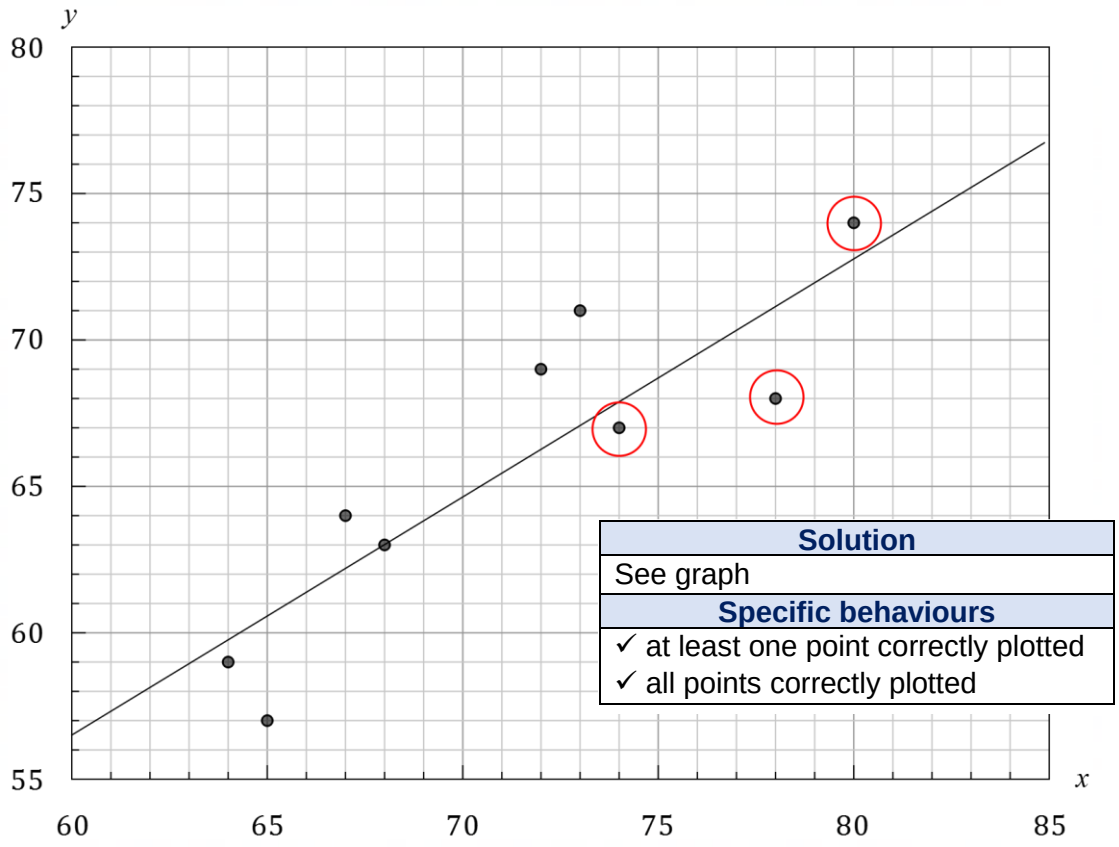
See next page

Question 12 **(13 marks)**

The table below shows the life expectancy, in years, of females and males in nine countries in Oceania.

Country	Female (x)	Male (y)
Federated States of Micronesia	72	69
Kiribati	64	59
Marshall Islands	73	71
Nauru	65	57
New Caledonia	80	74
Palau	78	68
Papua New Guinea	68	63
Solomon Islands	74	67
Tuvalu	67	64

(a) On the scatterplot below, plot the three missing data points from the table. (2 marks)



Solution
See graph
Specific behaviours
✓ at least one point correctly plotted
✓ all points correctly plotted

(b) Determine the coefficient of determination between the variables and interpret its value in the context of the question. (2 marks)

Solution
$r^2 = 0.7947 \approx 0.79$ 79% of the variation in the male life expectancy can be explained by the variation in the female life expectancy.
Specific behaviours
✓ correct coefficient (to at least 2 dp)
✓ correct interpretation

See next page

(c) State the correlation coefficient between the variables and use its value to comment on the strength of the linear association between female and male life expectancy for these countries. (2 marks)

Solution
$r = 0.891$ The linear association between the variables is strong.
Specific behaviours
✓ correct value of r ✓ states association is strong

(d) Determine the equation of the least-squares line to model the relationship between the variables and draw this line on the scatterplot. (3 marks)

Solution
$y = 0.877x + 3.294$
Specific behaviours
✓ correct equation (coefficients to at least 2 dp) ✓ any reasonable line of best fit ✓ ruled lined through $(60, 56)$ & $(85, 78)$

(e) The life expectancy of a female from Fiji is 70. Predict, to the nearest year, the life expectancy of a male from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

Solution
$\hat{y}(70) = 64.7 \approx 65$ With strong correlation and the age lying within the range of data, it is reasonable to assume the prediction is valid.
Specific behaviours
✓ prediction ✓ notes valid, with at least one reason

(f) The life expectancy of a female Australian is 86. Predict, to the nearest year, the life expectancy of a male Australian and comment on any factors that affect the validity of your prediction. (2 marks)

Solution
$\hat{y}(86) = 78.7 \approx 79$ Despite the strong correlation, this prediction involves extrapolation and should be treated with caution.
Specific behaviours
✓ prediction ✓ notes dangers of extrapolation

See next page

Question 13 **(7 marks)**

Anna had a bank account that paid no interest. At the start of the year her account balance was $\$3450$, and at the end of the first week and every week thereafter she withdrew $\$75$.

(a) Calculate the balance of Anna's account after 4 weeks. (1 mark)

Solution
$A_4 = \$3850 - 4 \times \$75 = \$3150$
Specific behaviours
✓ correct balance

Let the balance in Anna's account at the end of the n^{th} week be A_n.

(b) Deduce a rule for A_n and hence determine the balance of Anna's account after 12 weeks. (2 marks)

Solution
$A_n = 3850 - 75n$ $A_{12} = 3850 - 75 \times 12$ $= \$2550$
Specific behaviours
✓ correct rule (accept recursive or n^{th} term) ✓ correct balance

(c) For how many weeks was Anna able to withdraw $\$75$? (1 mark)

Solution
46 weeks, since $A_{46} = 0$.
Specific behaviours
✓ correct number of weeks

In the same year, Anna's friend Ben had a similar account. The balance B_n of his account at the end of the n^{th} week was given by the recurrence relation $B_{n+1} = B_n - 60,\ B_0 = 3030$.

(d) Determine the balance of Ben's account after 12 weeks. (1 mark)

Solution
$B_{12} = \$2310$
Specific behaviours
✓ correct balance

(e) At the end of one week during the year, the balance of Anna's account was identical to that of Ben's. Determine which week this was and the balance of both accounts at that time. (2 marks)

Solution
$A_{28} = B_{28} = 1350$ At the end of the 28^{th} week, when their balances were $\$1350$.
Specific behaviours
✓ correct week ✓ correct balance

See next page

Question 14 **(7 marks)**

A business bought a mainframe computer valued at $\$95\,000$. The value of the computer depreciated by 35% each year.

(a) By how much did the value of the computer depreciate during the first year and what was its value one year after it was bought? (2 marks)

Solution
$0.35 \times \$95\,000 = \$33\,250$ $V_1 = \$95\,000 - \$33\,250 = \$61\,750$
Specific behaviours
✓ depreciation amount ✓ value after one year

(b) Deduce a recursive rule for V_n, the value of the computer after n years. (2 marks)

Solution
$V_{n+1} = 0.65V_n, \quad V_0 = 95\,000$
Specific behaviours
✓ indicates correct multiplier ✓ correct rule with initial term

(c) Calculate the value of the computer after 4 years. (1 mark)

Solution
$V_4 = \$16\,958.09$
Specific behaviours
✓ correct value (accept reasonable rounding)

(d) During which year does the value of the computer first depreciate by less than $\$1000$? Justify your answer. (2 marks)

Solution
$V_8 = 3027.13, \quad V_9 = 1967.63, \quad V_{10} = 1278.96$ By observing terms of the sequence, the annual depreciation will first be less than $\$1000$ during the 10^{th} year.
Specific behaviours
✓ indicates appropriate reasoning ✓ correct year, with reasoning

See next page

Question 15 **(10 marks)**

An industrial chemist varied the amount of accelerant (a grams) used when making an epoxy resin and recorded the time taken (t seconds) for the resin to set. The results are shown below.

a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
t	24.1	19.2	19.3	21.8	15.7	19.2	14.8	17.7	15.0	12.3

The chemist suspected that a linear association might exist between the variables and calculated the correlation coefficient $r_{at} = -0.81$.

(a) After seeing this value of the correlation coefficient, the chemist said to their assistant "it looks like there is a strong and negative linear association between the variables". Explain this interpretation of the coefficient. (2 marks)

Solution
Since r is close to -1, the strength of the linear association is strong. Since $r < 0$, the direction of the linear association is negative.
Specific behaviours
✓ explains strong ✓ explains negative

The chemist also noted that the least-squares line for the data was $\hat{t} = 25.78 - 0.896a$ and used it to calculate nine residuals for the linear model as shown below, rounded to one decimal place.

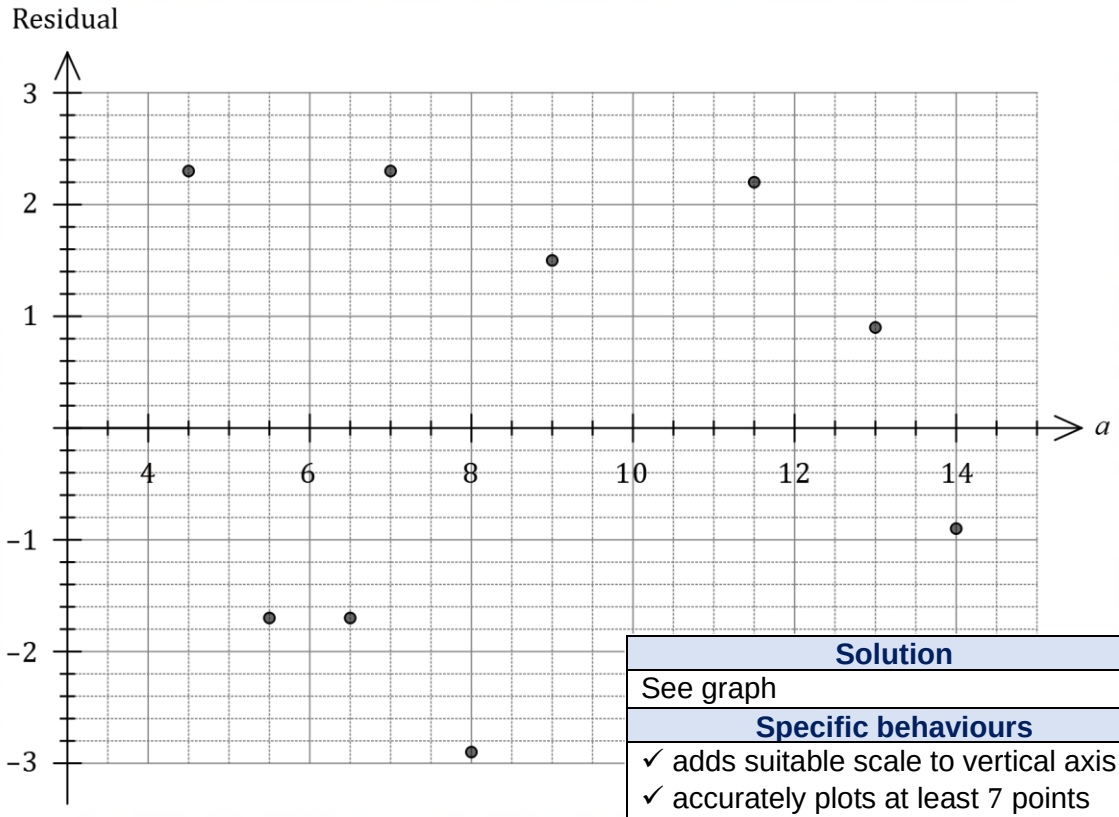
a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
Residual	2.3	-1.7	-1.7	**2.3**	-2.9	1.5	-2.0	2.2	0.9	-0.9

(b) Show how the residual of -2.9 was calculated and determine the residual associated with 7.0 grams of accelerant. (3 marks)

Solution
$\hat{t}(8.0) = 25.78 - 0.896(8.0) = 18.6, \quad 15.7 - 18.6 = -2.9$ $\hat{t}(7.0) = 25.78 - 0.896(7.0) = 19.5, \quad 21.8 - 19.5 = 2.3$
Specific behaviours
✓ calculates $\hat{t}(8.0)$ and subtracts from 15.7 ✓ shows calculation for $\hat{t}(7.0)$ ✓ calculates missing residual

See next page

(c) Construct a residual plot for the data on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ adds suitable scale to vertical axis
✓ accurately plots at least 7 points
✓ accurately plots all points

(d) Does the residual plot support the chemist's suspicions that a linear model fits the data? Explain your answer. (2 marks)

Solution
The residual plot supports the linear model as no pattern is evident in the residuals.
Specific behaviours
✓ states plot supports linear model
✓ states no pattern evident in residuals

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Question 16 **(6 marks)**

The mass of a small puppy was measured as 625 g when it was one week old. A week later its mass had increased by 50 g.

(a) Assuming that the weekly mass of the puppy can be modelled by an arithmetic sequence, predict the mass of the puppy when it is 8 weeks old. (2 marks)

Solution
$T_n = 625 + 50(n-1)$ $T_8 = 625 + 50 \times (8-1)$ $= 975$ g
Specific behaviours
✓ indicates appropriate method ✓ correct mass

(b) Assuming that the weekly mass of the puppy can be modelled by a geometric sequence, predict the mass of the puppy when it is 8 weeks old. (3 marks)

Solution
$r = \dfrac{625+50}{625} = 1.08$ $T_n = 625(1.08)^{n-1}$ $T_8 = 625(1.08)^{8-1}$ $= 1071$ g
Specific behaviours
✓ indicates common ratio ✓ indicates appropriate method ✓ correct mass

(c) Comment on the usefulness of these models as the puppy gets older. (1 mark)

Solution
Not very useful, since both models have the mass of the puppy increasing for ever, yet all dogs reach their adult weight after a year or two.
Specific behaviours
✓ sensible comment that notes models eventually not useful

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Question 17 **(8 marks)**

The annual number of mobile phone subscriptions and new cars sold in New South Wales, as collated by a researcher, are shown in the table below.

Year	2012	2013	2014	2015	2016	2017	2018
Subscriptions (s, in millions)	10.3	10.5	10.7	11.0	11.2	11.4	11.7
New cars (c, in thousands)	321	329	336	342	347	353	359

The researcher wanted to identify whether new car sales in New South Wales could be predicted from mobile phone subscriptions.

(a) Quantify the strength of the linear association between the variables s and c. (1 mark)

Solution
$r = 0.994$
Specific behaviours
✓ calculates r

(b) Determine the equation of the least-squares line that can be used to predict c from s. (2 marks)

Solution
$\hat{c} = 26.48s + 50.46$
Specific behaviours
✓ correct coefficients ✓ uses given variables

(c) Use the least-squares line to predict the number of new car sales in another Australian state that had 11.5 million mobile phone subscriptions, and comment, with reasons, on the validity of your prediction. (3 marks)

Solution
$\hat{c} = 26.48(11.5) + 51.46 = 355$ Sales predicted to be 355 thousand cars. This prediction is not valid as the line is derived from sales of cars and smartphones in NSW. In another state, it is unlikely that the variables will have the same association as in NSW.
Specific behaviours
✓ correct prediction, noting units ✓ states, with reasoning, that prediction not valid ✓ supplies reason that prediction not valid

(d) Describe a possible non-causal explanation for the observed association between mobile phone subscriptions and new cars sold. (2 marks)

Solution
The association is likely due to a common response to a third variable - the population of the state. As the population increases, so there will be more people to buy cars and mobile phone subscriptions.
Specific behaviours
✓ identifies a confounding variable such as population (*do not accept time*) ✓ explains common response to confounding variable

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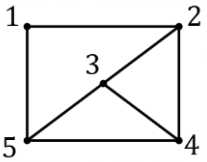
Question 18 **(9 marks)**

The adjacency matrix for the connected planar graph P is $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.

(a) Determine, with justification, the number of faces that P has. (3 marks)

Solution
 P has 4 faces.
Specific behaviours
✓ draws P, correct v and e ✓ draws P in the plane ✓ states number of faces

Alternative Solution
Vertices $v = 5$ (number of matrix rows) and edges $e = 7$ (sum of elements above matrix diagonal). P is a connected planar graph, so using Euler's formula then $5 + f - 7 = 2$ and so $f = 2 + 7 - 5 = 4$. P has 4 faces.
Specific behaviours
✓ states number of vertices and edges ✓ uses Euler's formula ✓ states number of faces

(b) Use elements from the adjacency matrix to explain why P is a simple graph. (3 marks)

Solution
Elements on the leading diagonal are all 0 and so there are no loops. All other elements in the matrix are 0 or 1 and so there are no multiple edges.
Specific behaviours
✓ states no loops and no multiple edges ✓ uses diagonal elements to justify no loops ✓ uses other elements to justify no multiple edges

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(c) Ore's theorem states that a simple graph with n vertices is Hamiltonian if, for every pair of distinct vertices V_a and V_b which are not adjacent, the sum of the degrees of V_a and V_b is greater than or equal to n. Use Ore's theorem to show that P is Hamiltonian. (3 marks)

Solution
P has three pairs of vertices that are not adjacent: V_1 and V_3 with degrees 2 and 3 respectively. V_1 and V_4 with degrees 2 and 3 respectively. V_2 and V_5 with degrees 3 and 3 respectively. The sum of each pair is clearly greater than or equal to $n = 5$, the number of vertices, and so P is Hamiltonian. (*NB Using adjacency matrix, non-adjacent pairs identified by* 0 *elements not on leading diagonal, and degree is sum of elements in row.*)
Specific behaviours
✓ identifies the three pairs of vertices that are not adjacent ✓ states degrees of pairs of vertices ✓ states sum of pairs is at least 5 and draws conclusion

See next page

Question 19 **(8 marks)**

Joe plans to invest $\$88\,000$ in an account that pays interest of 0.65% per month. At the end of each month, just after interest is added to the account, he will withdraw $\$330$. The balance of his account, a_n, after n withdrawals can be modelled by the recurrence relation

$$a_{n+1} = 1.0065a_n - 330, \qquad a_0 = 88\,000.$$

(a) Determine the balance of the account after 6 withdrawals have been made and describe how the balance has changed since the account was opened. (2 marks)

Solution
$a_6 = \$89\,475.80$ $\Delta = \$89\,475.80 - \$88\,000 = \$1475.80$ Account balance has increased by $\$1475.80$.
Specific behaviours
✓ calculates a_6 ✓ states balance has increased and amount of increase

(b) Calculate the total withdrawn from the account after 6 withdrawals, and hence show that the total interest paid into the account over this time is $\$3455.80$. (2 marks)

Solution
Total withdrawn: $6 \times \$330 = \$1980$. Hence $I = \$1475.80 + \$1980 = \$3455.80$.
Specific behaviours
✓ calculates total withdrawn ✓ shows how to derive total interest

(c) The balance of Joe's account will first exceed $\$95\,000$ after the k^{th} withdrawal. Determine the value of k and state the balance of the account at this time. (2 marks)

Solution
$k = 27, \qquad a_{27} = \$95\,117.23$
Specific behaviours
✓ value of k ✓ value of a_k

(d) If, after making the 6^{th} withdrawal, Joe then changed the amount he withdrew each month to $\$590$, explain how this would change the way the account balance grew in the future? Justify your answer. (2 marks)

Solution
$a_{n+1} = 1.0065A_n - 590, \qquad a_0 = 89\,475.80$ For this new sequence, the account balance doesn't grow but starts to decrease, as seen by $a_1 = \$89467.39$, which is $\$8.41$ less than a_0.
Specific behaviours
✓ shows modified recursive rule ✓ uses terms of new sequence to explain balance now decreases

End of questions

Supplementary page

Question number: _____